The Multiphoton Interaction of "Λ" Model Atom and Two-Mode Fields

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Abstract

Starting from the C.T.Lee's criterion on non-classical effects in two-mode fields,the author of this paper have studied the system of two-mode fields interacting with atom by means of multiphotons, discussed the non-classical statistic quality of two-mode fields with interaction. Through mathematical calculation, we've come to realize some new rules of non-classical effects of two-mode fields which evolue with time.

1 Introduction

The non-classical effects of field is very interesting topic in quantum optics,and for a long time the interaction of atom with field,and its quantum statistic quality have been paid extensive attention to.Since E.T.Jaynes and F.W.Cummings put forward an ideal model of interaction of two-level atom and one-mode field[1] strictly resolved by pure quantum methods, people have done lots of research on J-C Model in quantum optics these years, for example,interactions of two-level atom with one photon[2],two- photons[3] and multiphotons[4−8],etc. Because one atom may often have multiple levels,and plenty of experiments require the consideration of a third level, people have naturally proceeded the J-C Model to the third level and discussed the interaction of three-level atom with one field.(These are called broad J-C Model),one photon process and multiphoton process,and as a result discovered many non-classical phenomena with different quanlities,for instance,those of revival-collapse as well as squeezing of field and antibunching,etc.[9−11]
The significance of studying J-C Model and broad J-C Model exist in the realization of the respective quantum dynamics qualities of atom and field when they interact. Though people have done a great deal of research on J-C Model, their research is restricted to the resonance and non-resonance of one or two-photon, not covering the function of $K(>2)$ photons. We have already discussed the quantum statistic quality of multiphoton process[12].By adopting the broad J-C Model and using density operator, we have obtained the mean-photon number value in this paper,and then discussed the quantum statistic quality of interaction of three-level atom with two-mode fields according to the criterion on non-classical effects put forward by C.T.Lee and consequent found some new evolution rules.

2 Theoretical model

Let's think about the system of the interaction of "Λ" atom with two-mode fields,$| a >$ is given as common upper level,$| b >$ and $| c >$ are given as lower levels, the state vector of atom are taken from

$$| a >= \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} | b >= \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} | c >= \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$| a >$ and $| b >$, $| a >$ and $| c >$ is related with mode 1 and mode 2, between the $| b >$ and $| c >$ is forbiddenness of a transition. On condition of resonance ($\omega_a - \omega_b = k_1\Omega_1, \omega_a - \omega_c = k_2\Omega_2$) and RWA, the Hamiltonian of the system is expressed as

$$H = \hbar(H_0 + H_{int}) \tag{1}$$

the free part of which is

$$H_0 = \omega_a s_{11} + \omega_b s_{22} + \omega_c s_{33} + \sum_{i=1}^{2} \Omega_i a_i^+ a_i \tag{2}$$

and the interacting part is

$$H_{int} = \sum_{i=1}^{2} \lambda_i (s_{1,i+1} a_i^{k_i}) + h.c \tag{3}$$

where,$s_{1,i+1}$ are the transition operators of atom,s_{ii} (i=1,2,3) is the level projection oprators of atom,a_i, a_i^+ and Ω_i(i=1,2) are the annihilation and creation operators, and angle frequencies of the mode i field, λ_i is the coupling constant of atom and field,ω_a, ω_b and ω_c are the correspondence frequencies of $| a >, | b >$ and $| c >$,k_1 and k_2 are the photon number of absorption or emission in transition process of atom between $| a >$ and $| b >, | a >$ and $| c >$.
Because H_0 and H_{int} are the motion constant, then

$$[H_0, H_{int}] = [H_0, H] = [H_{int}, H] = 0 \tag{4}$$

so,in the interacting picture, there exists

$$H_{int}^I = e^{iH_0t} H_{int} e^{-iH_0t} = H_{int} \tag{5}$$

evolution operator of time in interacting picture is

$$U(t) = exp(-iH_{int}^I t) \tag{6}$$

where the factors of density matrix $\rho(0)$ of the initial system of atom and field is given, the average value of physical quantities of the system can be obtained from

$$< Q >= T_r[\rho(t)Q] = T_r[U(t)\rho(0)U^+(t)Q] \tag{7}$$

If t=0,the atom is in the common upper state $| a >$, the two-mode fields are in coherent states $| \alpha_1 >$ and $| \alpha_2 >$, and possese their respective mean photon numbers $\overline{n_1} =| \alpha_1 |^2$ and $\overline{n_2} =| \alpha_2 |^2$, the initial density matrix of atom and fields of the system can be shown as

$$\rho(0) = \begin{pmatrix} \rho_f(0) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{8}$$

where the initial density operator of field

$$\rho_f(0) = \sum_{m_1,m_1'} \frac{\alpha_1^{m_1}\alpha_2^{m_2}\alpha_1^{m_1'}\alpha_2^{m_2'}}{(m_1!m_2!m_1'!m_2'!)^{1/2}} exp[-\overline{n}_1 - \overline{n}_2] \mid m_1, m_2 >< m_1', m_2' \mid \tag{9}$$

the mean photon numbers of two-mode fields can be gained from(6)-(9)

$$< n_1 >= \overline{n}_1 + \sum_{n_1,n_2} \lambda_1^2 k_1 \frac{(n_1+k_1)!}{n_1!} \frac{sin^2 ut}{u^2} p(n_1, n_2) \tag{10}$$

$$< n_2 >= \overline{n}_2 + \sum_{n_1,n_2} \lambda_2^2 k_2 \frac{(n_2+k_2)!}{n_2!} \frac{sin^2 ut}{u^2} p(n_1, n_2) \tag{11}$$

and

$$< n_1^2 >= \overline{n}_1(\overline{n}_1 + 1) + \sum_{n_1,n_2} \lambda_1^2 (2n_1 + k_1) k_1 \frac{(n_1+k_1)!}{n_1!} \frac{sin^2 ut}{u^2} p(n_1, n_2) \tag{12}$$

$$< n_2^2 >= \overline{n}_2(\overline{n}_2 + 1) + \sum_{n_1,n_2} \lambda_2^2 (2n_2 + k_2) k_2 \frac{(n_2+k_2)!}{n_2!} \frac{sin^2 ut}{u^2} p(n_1, n_2) \tag{13}$$

$$< n_1 n_2 >= \overline{n}_1\overline{n}_2 + \sum_{n_1,n_2} \left[\lambda_1^2 k_1 n_2 \frac{(n_1+k_1)!}{n_1!} + \lambda_2^2 k_2 n_1 \frac{(n_2+k_2)!}{n_2!} \right] \frac{sin^2 ut}{u^2} p(n_1, n_2) \tag{14}$$

where

$$u = \left[\lambda_1^2 \frac{(n_1+k_1)!}{n_1!} + \lambda_2^2 \frac{(n_2+k_2)!}{n_2!} \right]^{1/2}$$

$$p(n_1, n_2) = \frac{\overline{n}_1^{n_1}\overline{n}_2^{n_2}}{n_1!n_2!} exp(-\overline{n}_1 - \overline{n}_2)$$

3 The Statistic quality of field

In general,the non-classical effects of field include squeezing state,antibunching and sub-Poisson distribution,which have been experimented. In 1990,C.T.Lee put forward the definition of second-order correlation function at zero time in identical and different modes in the two-mode field theory.The definition goes as:

$$c_i^{(2)}(0) = < n_i^{(2)} > - < n_i >^2 \; (i = 1, 2) \tag{15}$$

$$c_{12}^{(2)}(0) = < n_1 n_2 > - < n_1 >< n_2 > \tag{16}$$

where $< n^{(2)} >$ is the second factorial moment of the photon number.Therefore we have $C_i^{(2)}(0) = 0$ for a coherent radiation, since it has a Poisson distribution of photon numbers, when $C_i^{(2)}(0) > 0$, we call it intramode photon bunching, which is always true for classical radiation; and, in contrast,we have intramode photon antibunching when $C_i^{(2)}(0) < 0$,which is possible only for non-classical fields.In analogy to Epn.(15), we call it intermode photon bunching, if we have $C_{12}^{(2)} > 0$,or intermode photon antibunching if $C_{12}^{(2)}(0) < 0$. C.T.Lee also put forward the criterion of second order non-classical effects about two-mode radiation fields,the criterion is:

$$D_{12}^{(2)} = < n_1^{(2)} > + < n_2^{(2)} > -2 < n_1 n_2 >< 0 \tag{17}$$

the smaller the $D_{12}^{(2)}$, the deeper the non-classical degree of two-mode radiation fields.
Now let's discuss the evolution rules of $C_1^{(2)}(0), C_2^{(2)}(0), C_{12}^{(2)}(0)$ and $D_{12}^{(2)}$ in case of given initial mean photon numbers $\overline{n}_1$ and $\overline{n}_2$, transition photon numbers k_1 and k_2. Let $\lambda_1 = \lambda_2 = \lambda, \varphi_1 = \varphi_2 = 0$,we can see:
(a) When initial strength of two-mode fields is weak,$\overline{n}_1 = \overline{n}_2 = 1$.the transition photon number $k_1 = k_2 = 1$ or 10, the evolution curves of $C_1^{(2)}(0)$ and $C_2^{(2)}(0)$ are the same. In $k_1 = k_2 = 1$ process, there is cyclic fluctuation of intramode photon antibunching and bunching, and there is always intermode photon antibunching, and begin $D_{12}^{(2)} > 0$, then there are non-classical effects, which become deeper and deeper as time increases,whth fluctuation being weak, getting close to oblique line to the right below. In $k_1 = k_2 = 10$ process, there is not intramode photon antibunching, but there is intermode photon antibunching for $C_{12}^{(2)}(0)$, and there is $D_{12}^{(2)} < 0$ from beginning to end, the evolution curves are irregular.
(b) When initial strength of the field increases,$\overline{n}_1 = \overline{n}_2 = 10$, and $k_1 = k_2 = 1$, there are alway intramode photon antibunching, the revival- collapse phenomena of the evolution curves are obvious, there is intermode photon antibunching for $C_{12}^{(2)}(0)$,and $D_{12}^{(2)} < 0$,the evolution curves are steeper compared with(a). When $k_1 = k_2 = 10, C_1^{(2)}(0)$ and $C_2^{(2)}(0)$ are both bigger than zero,but there is intermode photon antibunching for $C_{12}^{(2)}(0)$,and $D_{12}^{(2)} < 0$, the evolution curves fluctuate

faster, compared with the above.
(c)When initial strength of the field $\overline{n}_1$ and $\overline{n}_2$ are constant, and the transition photon numbers is changed. If $\overline{n}_1 = \overline{n}_2 = 1, k_1 = 1, k_2 = 10$, there is alway intramode photon antibunching for $C_1^{(2)}(0)$, but not for $C_2^{(2)}(0)$, there is alway intermode photon antibunching for $C_{12}^{(2)}(0)$, there is $D_{12}^{(2)} < 0$ from start to finish, all amplitudes of evolution curves increase. If $k_1 = 10, k_2 = 1$,the evolution curves of $C_1^{(2)}(0)$ are pictured as $C_2^{(2)}(0)$, curves of $C_2^{(2)}(0)$ as $C_1^{(2)}(0)$, curves of $C_{12}^{(2)}(0)$ and $D_{12}^{(0)}$ are as the above. When the transition photon number is given, the initial strength of the field is changed, if $k_1 = k_2 = 1, \overline{n}_1 = 0.1, \overline{n}_2 = 1$, the evolution curves of $C_1^{(2)}(0)$ are pictured as $C_1^{(2)}(0)$ in (a)($\overline{n}_1 = \overline{n}_2 = k_1 = k_2 = 1$),but the wavy curves are parallelly shifted down, and the amplitudes increase a little, and $C_2^{(2)}(0)$ are pictured as $C_1^{(2)}(0)$ in (a). If $\overline{n}_1 = 1, \overline{n}_2 = 0.1$,the evolution curves of $C_1^{(2)}(0)$ and $C_2^{(2)}(0)$ are reversed compared with the above. There are antibunching for $C_{12}^{(2)}(0)$ and non-classical effects for $D_{12}^{(2)}$.

4 conclusion

The result of the paper continues to show that the non-classical effects of two-mode fields interacting with "Λ" atom by means of multiphotons are not only related to the initial strength of two-mode fields,but also absorption or emission the number of photons in the transition process between atomic levels.In the same time an interesting phenomenon of mode-competition exists,under identical conditions, two modes have identical status in the mode-competition, and $C_1^{(2)}(0)$ and $C_2^{(2)}(0)$ reveal identical evolution rules. Different conditions ,namely identical number of photons transition between atomic levels but different initial strengths of two-mode fields,or different number of photons transition between atomic levels but identical initial strength of two-mode fields will all lead to the generation of the phenomenon of mode- competition. On the other hand, the more the transition photons. the more disadvantageous they are to register where fields enter non-classical effects.

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